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Neighborhood Systems of Identity of A Semitopological Lattice Ordered Group

Abstract

In view of Theorem 1 [2] which states as fallows: Let a be a fixed element of a Semitopological lattice ordered group *L*. Then the mapping

$$\gamma_a : x \to xa$$
$$L_a : x \to ax$$

of *L* onto *L* are homeomorphisms of *L*.

It follows that if one knows a fundamental system of neighborhoods of the identity of a semitopological lattice ordered group, then one can find a fundamental system of neighborhoods of any other point by translation.

In the sequel, we show that actually the topology of a semitopological lattice ordered group is completely determined by a fundamental system of neighborhoods of its identity. More precisely, we have the following theorem 1 :

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Introduction

The relation between topological lattice ordered group and semitopological lattice ordered group has been established in [2].

Aim of the study

In this paper our purpose is to study fundamental system of neighborhoods of its identity.

Bases of the Semitopological Lattice Ordered Group

The identity of semitopological lattice ordered multiplicative group will be denoted by *e* and that of an additive group by *o*.

Theorem 1

If {*U*} is a fundamental system of open neighborhoods of *e* in a semitopological lattice ordered group *L*, then {*x U*} and {*U x*}, where *x run over* L and *U* over {*U*}, form bases of the topology of *L*.

Conversely, let a filter base {*U*} be given so that each *U* contains *e* and for each U $x \in U$

There exist *V* and *W* in {*U*} such that $xV \subset U$ and $Wx \subset U$ Then there exists a topology *u* on *L* so that *L*, endowed with *u*, is a semitopological lattice ordered group.

Proof

Let $a \in L$ and let W be an open neighborhood of a. Since $l_a^{-1} : x \to a^{-1}x$ is a homeomorphism in *x* (Thorem1 [2]).

$l_a^{-1} : (W) = a^{-1}W$ is an open set containing e and hence there exists a *U* in {*U*} such that $U \subset a^{-1}W.$ This implies $aU \subset W$, which proves that {*xU*} is a base of the topology on *L*.

Similar arguments show that {*Ux*} in also a base.

Conversely, let $\mathfrak{U}$ denote the family of all finite intersection of members in {*U*}. Than $\mathfrak{U}$ is a nonempty family of U, each of which contains *e*.

Furthermore, for any $\tilde{U} = \bigcap_{i=1}^{n} U_i$

Hence $x\tilde{U} = x\left(\bigcap_{i=1}^{n} U_i\right) = \bigcap_{i=1}^{n} xU_i$

for any $x \in L$. And if $x \in \bigcap_{i=1}^{n} U_i$, then there exists aV_i for each $i, \leq i \leq n$ such that $xV_i \subset U_i$ and hence,

$$\bigcap_{i=1}^{n} xV_i = x\left(\bigcap_{i=1}^{n} V_i\right) \subset \bigcap_{i=1}^{n} U_i .$$

This shows that the family $\overline{\mathfrak{U}}$ also satisfies the conditions assumed for the filler base {U}. By the definition of a subbase, the family of finite intersection of the family {*xU*}, where *x* runs over *L* and *U* over {*U*}, forms a base of the topology *u* on *L*.

Now if,

$y \in \bigcap_{i=1}^{n} x_i U_i$, then $x^{-1} y \in U_i$ for each *i*, $1 \leq i \leq n$ and hence by assumption, there exist a $V_i \in \{U\}$ such that $x_i^{-1} y V_i \subset U_i$, or $yV_i \subset x_i U_i$.

This shows that

$$y\left(\bigcap_{i=1}^{n} V_i\right) = \bigcap_{i=1}^{n} yV_i \subset \bigcap_{i=1}^{n} x_i U_i$$

Therefore, $\{y\tilde{U}\}$, Where $\tilde{U}$ runs over $\overline{\mathfrak{U}}$ forms a fundamental system of open neighborhoods of *y* for each $y \in L$ Similarly it can be shown that $\{\tilde{U}y\}$ is also a fundamental system of neighborhoods of *y*.

Now to complete the proof, we have to show that *L*, endowed with *u*, is a semitopological lattice ordered group. Consider the mapping

$$g_1 : (x, y) \to xy$$

Assume first that *x* is fixed, and Let $\tilde{U}$ be any member of $\overline{\mathfrak{U}}$ Then $xy\tilde{U}$ is a member of a fundamental system of neighborhoods of *xy*.

Since $y \in y\tilde{U}$ and $y\tilde{U}$ is a *u*-neighborhoods of *y* as shown in the previous paragraph.

$$x\left(y\tilde{U}\right) \subset xy\tilde{U}$$

This proves the continuity of g_1 in *y* while *x* is kept fixed. Similarly, we can prove the continuity of g_1 in *x* by considering $\tilde{U}xy$ as a neighborhood of *xy*. This completes the proof.

A similar theorem, true for semitopological lattice ordered groups, will be proved later on.

Endnotes

1. *G. Birkhoff: Lattice theory, AMS Publication, reprinted 1984, p. 25, 248.*
2. *K. Parhi and P. Kumari Semitopological Lattice ordered group; International J. of Creative Research thoughts, Vol. 6, Issue 2, April 2018, 374-375.*